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$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Exam one MTH 221 , Summer 2022

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Score = $\frac{48}{48}$ Excellent

QUESTION 1. (16 points)

(i) Let $A = \begin{bmatrix} 3 & 2 & 6 \\ 6 & 7 & 8 \\ 9 & 0 & 4 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 6 & 2 \\ 0 & 1 & 2 & 6 \\ 1 & 1 & 6 & 2 \end{bmatrix}$, and $C = AB$. Then the (4, 3)-entry of C (i.e., the number in C that is in the 4th row and 3rd column) is

3×4

$$6\begin{bmatrix} 3 \\ 6 \\ 9 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 7 \\ 0 \\ 0 \end{bmatrix} + 6\begin{bmatrix} 6 \\ 8 \\ 4 \\ 3 \end{bmatrix}$$

(a) 18 (b) 6 (c) 78 (d) 26

(ii) Let A, B, and C as in (i). Let A_1, A_2, A_3 be the first, the second and the third column of A, respectively. Let C_4 be the 4th column of C. We know $C_4 = b_1A_1 + b_2A_2 + b_3A_3$. Then the values of b_1, b_2 and b_3 are

(a) $b_1 = 2, b_2 = 6, b_3 = 2$ (b) $b_1 = 0, b_2 = 0, b_3 = 3$ (c) $b_1 = 9, b_2 = 0, b_3 = 4$ (d) $b_1 = 6, b_2 = 2, b_3 = 6$

(iii) Let $D = span\{(1, 1, -1, 1), (-1, -1, 1, 2), (2, 2, -2, -2), (-2, -2, 2, 1)\}$. Then a basis, B, for D is

(a) $B = \{(1, 1, -1, 1), (-1, -1, 2, 2), (2, 2, -2, -2), (-2, -2, 3, 1)\}$

(b) $B = \{(1, 1, -1, 1), (-1, -1, 2, 2), (-2, -2, 3, 1)\}$

(c) $B = \{(1, 1, -1, 1), (-1, -1, 2, 2), (2, 2, -2, -2)\}$

(d) $B = \{(1, 1, -1, 1), (0, 0, 0, 3)\}$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4, 4, -4, 4

(iv) Let D as in (iii). One of the following points belongs to D. 4(1, 1, -1, 1) (0, 0, 0, 3)

(a) (2, 2, -1, 9) (b) (-1, -1, -1, 2) (c) (3, -3, -3, 5) (d) (4, 4, -4, 0.27)

(v) One of the following is a subspace of R^4

(a) $D = \{a(1, 1, -1, 1) + b(2, 1, 0, 3) \mid a \in R \text{ and } b \geq 0\}$ x

(b) $D = \{(a, b, c, d) \mid a + b = 0 \text{ and } c + d = 0\}$

(c) $D = \{(a,\ a^2b,\ 0,\ b) \mid a, b \in R\}$ X

(d) $D = \{(a,\ a + b,\ 1,\ 0) \mid a, b \in R\}$ x

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(vi) Let $T : R^2 \to R$ be an R-homomorphism (linear transformation) such that $T(2, 0) = -4$ and $T(-6, 3) = 9$. Then T(2, 3) =

T(2,3) =

(a) 5 (b) -7 (c) -1 (d) -5

(vii) Given $D = \{(a - 2b + c,\ 0,\ -2a + 4b - 2c,\ -a + 2b - c) \mid a, b, c \in R\}$ is a subspace of R^4. Then $dim(D) =$

(a) 1 (b) 2 (c) 3 (d) 4

(1, 0, -2, -1) (-2, 0, 4, 2) (1, 0, -2, -1)

(viii) One of the following is a linear transformation.

(a) $T : R \to R^2$ such that $T(a) = (a, -3a)$ (b) $T : R^2 \to R^2$ such that $T(a, b) = (ab, a + b)$ x

(c) $T : R^2 \to R^2$ such that $T(a, b) = (a + 2b,\ 3 + b)$ x (d) $T : R^2 \to R$ such that $T(a, b) = a^2 - b^2$ x

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

QUESTION 2. Let $T : R^4 \to R^3$ such that $T(a,b,c,d) = (a + c + d,\ 0,\ -a + b - c - d)$.

(i) (2 points) Convince me that T is a linear transformation.

by staring, T is a linear transformation because the output can be written as a linear combination of (a, b, c, d)

(ii) (2 points) Find the standard matrix presentation of T

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & -1 \end{bmatrix}$$

(iii) (4 points) Find all points in the domain of T such that $T(a,b,c,d) = (4,0,8)$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & -1 & 8 \end{array}\right] \quad R_1 + R_3 \to R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 12 \end{array}\right]$$

leading : a , free : b, c, d

$a + c + d = 4$, $4 - c - d = a$

$b = 12$

$S.S = \{(4-c-d, 12, c, d) \mid c, d \in \mathbb{R}\}$

(iv) (4 points) Find $Z(T) = Ker(T) = Null(T)$ and write it as a span of **INDEPENDENT** points.

$Z(T) = \{(c-d, 0, c, d) \mid c, d \in \mathbb{R})$

Span $Z(T)$ = Span$\{(1,0,1,0), (-1,0,0,1)\}$

(v) (4 points) Find Range(T) and and write it as a span of **INDEPENDENT** points.

Span of Output $= (1,0,-1), (0,0,1), (1,0,-1), (1,0,-1)$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{array}{l} -R_1 + R_3 \to R_3 \\ -R_1 + R_4 \to R_4 \end{array} \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{\} Dependent}$$

$\therefore$ Span$\{(1,0,-1), (0,0,1)\}$

QUESTION 3. Let $T : R^3 \to R^2$ such that $T(1,0,0) = (1,2), T(0,-2,0) = (-2,-4)$ and $T(0,0,3) = (3,6)$.

(i) (4 points) Find the standard matrix presentation of T.

$T(1,0,0) = (1,2)$

$T(0,1,0) = -\frac{1}{2}(T(0,-2,0) = -\frac{1}{2}(-2,-4) = (1,2)$

$T(0,0,1) = \frac{1}{3}(T(0,0,3) = \frac{1}{3}(3,6) = (1,2)$

$$\begin{array}{ccc} (1,0,0) & (0,1,0) & (0,0,1) \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

(ii) (4 points) Find a basis for $Z(T) = Ker(T) = Null(T)$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right] \xrightarrow{-2R_1+R_2 \to R_2} \begin{array}{ccc} a & b & c \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

basis $= \{(-1,1,0),(-1,0,1)\}$

$a+b+c=0$

$a=-b-c$

$S.S = \{(-b-c, b, c)\} = Span\{(-1,1,0),(-1,0,1)\}$

(iii) (2 points) Find $T(2,-4,6)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + -4\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 6\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

QUESTION 4. (i) (2 points) Convince me that $D = \{(a,\ b+1,\ a,\ b) \mid a, b \in R\}$ is not a subspace of R^4.

by using the origin method, by staring the origin is not in D (0,1,0,0) ∴ D is not a subspace of $\mathbb{R}^4$

(ii) (4 points) Find a basis for the subspace $D = \{(a,b,c,d) \mid a - 2c + 3d = 0$ and $-a + b + 2c - 2d = 0\}$

$a = 2c - 3D$

$b = \cancel{2c} + 2d + \cancel{2c} - 3D$

$b = \cancel{-2c} + 2D + \cancel{2c} - 3D$ $-D$

$b = 2d - 3D$

$= -D$

$D = \{(2C-3D, -D, C, D) \mid C, D \in \mathbb{R}\}$

Span $\{(2,0,1,0),(-3,-1,0,1)\}$

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ -3 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ -3 & -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{3R_1+R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ 0 & -1 & 1.5 & 1 \end{bmatrix}$$ independent ∴ basis $= \{(2,0,1,0),(-3,-1,0,1)\}$